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THE DEVELOPMENT OF THEORETICAL APPROACHES OF LINEARIZATION OF THE STATIC CHARACTERISTICS OF INFORMATION TOOLS

Theoretical studies that develop approaches linearization of the static characteristics of information tools. On the example of passive scanning the surface of the device open flows using the proposed method performed linearization of the static characteristics that allow for a margin of error of linearity, range of measured values and parameters of information tools.

Теоретичні дослідження розвиваючі методи лінеаризації статичних характеристик інформаційних засобів. На прикладі пасивного сканування поверхні пристрою відкритих потоків за допомогою запропонованого способу лінеаризації статичних характеристик, які враховують похибку лінійності, діапазон вимірюваних величин і параметрів інформаційних засобів.

Теоретические исследования, развивающие методы линеаризации статических характеристик информационных средств. На примере пассивного сканирования поверхности устройства открытых потоков с помощью предлагаемого способа линеаризации статических характеристик, которые учитывают погрешность линейности, диапазон измеряемых величин и параметров информационных средств.

The problem and its relation to the scientific and practical problems. Shredding poor iron ore in the process of enrichment is now the bottleneck in this technology, which is constantly expanding. Drum mills, iron ore milled poor, cost overruns electricity, grinding media, lining due to the lack of effective means of control of process parameters. Since the energy and material cost overruns are large enough, the modern technology of iron ore crushing the poor are not consistent with the basic provisions of the State scientific and technical program "Resource-saving technologies of new generation in the mining and metals sector." Therefore the subject of the article devoted to the solution of this problem is urgent. The material in this paper are obtained when the research theme "Computer Integrated System automatically adjusting the ratio of ore / water ball mill circulating load» (0105U008334).

Analysis of studies and publications. The problem of automatic control of iron ores by grinding the poor over the years have decided to both domestic and foreign scientists. Special attention was paid to information tools that have worked in difficult operating conditions, and usually do not provide the necessary accuracy. One of the reserves to improve the accuracy of measurement is to reduce the value of the error of the non-linearity of the static characteristic measuring means at its linearization. Linearization of the static characteristics of measuring instruments mainly carried out using the Taylor series or secant [1]. These methods, especially the first, admit significant errors. These and other methods, moreover, are based on the specific types of static characteristics, does not allow to use them in cases of obtaining arbitrary dependencies while providing predetermined performance accuracy. However, none of these problems are not solved.

Statement of the problem. The aim of this work is the development of theoretical approaches of linearization of the static characteristics of media, regardless of their type, and with a certain indicators of accuracy.

Presentation of the material and results. Analysis showed that the static characteristics which have a point of inflection are versatile because they contain all the others inherent easier dependencies elements. Therefore, the problem of linearization of the static characteristics will be considered on the curves with a point of inflection. General provisions of the linearization method of static characteristics can be summarized as follows. Let the curve 1 with a bend in *y=f(x)* is replaced by the straight line 2 so that it passes through the intersection point *b* and the end of the static characteristic (point *a*) (Fig. 1). Then errors in these points *δ(b)=δ(a)*=0, and the relative error in the interval *ba*

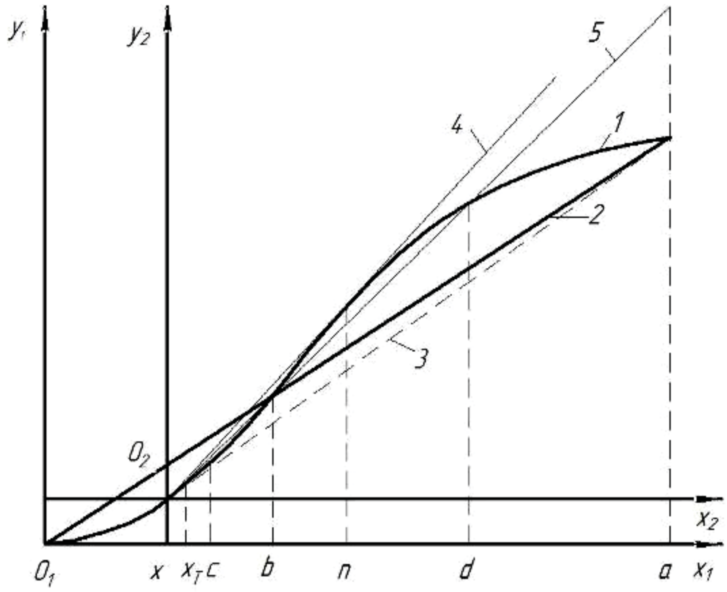

Fig. 1. Static characteristic of complementing lines in its linearization: 1-static characteristic; 2, 3, 4, 5 - supplementing lines

$$\delta(x) = (y - kx)/y = 1 - kx/y\,. \qquad (1)$$

By Rolle's theorem on the interval *ba* there is a maximum error of linearization, which is determined by the derivative *δ'(x)* at the point of maximum *δ(x)* is equal to zero. Under these conditions *y'=y/x*, i.e. slope of the tangent to the curve at the maximum relative error is the ordinate of the point to its abscissa. It is clear that this condition is satisfied only when the tangent 4 (Fig. 1) passes through the origin. On the other hand, if the linearization of an arbitrary curve straight conducted in a way that at a certain interval condition of Rolle's theorem for the function of the relative error of linearization, the point of the maximum error in this region tangent to the curve passes through the origin. Given this, the line which linearized be carried out at the condition of equality of errors in the points of maximum and *x*=0. If the curve *y=f(x)* has no inflection, the condition of Rolle's theorem does not hold. At the point whose tangent passes through the origin is a local maximum error of linearization is missing, but there is a maximum value of this function on the interval $x=0$, $x=b$ (Fig. 1), which corresponds to the intersection of the curve and the straight line. Therefore, in this case the line which linearized straight is on the condition that the errors at $x=0$ i $x=a$, which corresponds to the end value of the curve.

Consider a characteristic which decreases with increasing linearity argument and assumes the smallest value at the end according to. We transfer the coordinate system (Fig. 1) to the $O_2[x, f(x)]$. The presence of an inflection point allows you to get two zeros of the error function linearization at $x=c$ and $x=d$ (transversal 5 in Figure 1). The line which linearized can be carried out taking into account errors in the points $x_2=0$, $x_2=n-x$ and $x_2=a-x$, respectively, which are the highest in the intervals $[0, c-x]$, $[c-x, d-x]$, $[d-x, a-x]$ in the new coordinate system . The best option is when the error λ in these three points are the same. Having determined the relative values of the linearization errors in these points and making the expression, we obtain a system of three equations

$$\begin{aligned} &k/f_1'(x)-\lambda=1,\\ &k\left[\varphi(x)-x\right]/\left\{f\left[\varphi(x)\right]-f_1(x)\right\}+\lambda=1, .\\ &k(a-x)/\left[f_1(a)-f_1(x)\right]-\lambda=1, \end{aligned} \qquad (2)$$

where k - the slope; $\varphi(x)=x-n$; $f_1(x)$ - a function $f(x)$ in the new coordinate system, $f'_1(x)$ - its derivative.

The value of x in (2) will be considered as a parameter corresponds to the physical content of the problem. Then we have a system of three linear equations with two unknowns. The number of solutions of (2) depends on the intersection of the lines corresponding to the equations. Because line surely not the same because of differences in slope coefficients, the system (2) has a unique solution, or, in general, or it has not. According to the theorem of Kronecker-Capelli system of linear equations is compatible if and only if the rank of its main matrix equals the rank of the augmented matrix of the system of equations. According to equation (2) the main rank of the matrix A is equal to two. In order for the system to be compatible, it is necessary and sufficient that the rank of the augmented matrix B is also equal to two. Therefore, the determinant of B must be zero. This makes it possible to write

$$f_1'(x)=\left[f_1(a)-f_1(x)\right]/(a-x). \qquad (3)$$

Expression (3) has the following geometric interpretation: the equality of the three errors in these locations is only performed for the point x_T curve tangent where it passes through the end of the static characteristic (line 3 in Figure 1). If the system is moved to a point x in such a way that the condition $x>x_T$, then the tangent to the curve at this point will be above the end of the static characteristic.

The study found the growth of the relative error for a line passing through one of the points discussed by increasing its slope coefficient k of the initial value k_0. Found that with an increase in the slope coefficient k straight error

$\delta(0)$ and $\delta(a\text{-}x)$ increased linearly, and $\delta(a\text{-}x)> \delta(0)$. Based on the fact that when the error at k $x_2=n\text{-}x$ decreases the line which linearized must pass under the terms of equality of errors at the points $x_2=n\text{-}x$ and $x_2=a\text{-}x$, where $\delta(0)$ is less than $\delta(n\text{-}x)= \delta(a\text{-}x)$. With the growth of x within a certain range $\delta(0)$ will decrease, so that the limit value of x at which will be carried out according to the linearization of the condition is such that the tangent to the curve at this point is the line which linearized, that $\delta(0)=0$. Given the values of relative errors at these two points and the fact that $k= f'_1(x)$, after transformations we obtain the dependence of the limit values for $x=x_к$

$$(n-x)/\left[f_1(n)-f_1(x)\right]+(a-x)/\left[f_1(n)-f_1(x)\right]=2/f_1'(x). \tag{4}$$

Given the values of the relative error and the expression (4), after conversion to obtain the dependence of the error on the site of linearization [x_T, $x_к$]

$$\lambda=2/\{1+\left[f_1(a)-f_1(x)\right](n-x)/(a-x)\left[f_1(n)-f_1(x)\right]-1\}. \tag{5}$$

The slope of the line will be equal to the linearized

$$k=2/\{(a-x)/\left[f_1(a)-f_1(x)\right]+(n-x)/\left[f_1(n)-f_1(x)\right]\}. \tag{6}$$

For a period of [$x_к$, b] linearization is similar. However, only take into account that the error of the linearization at the point x_2=0 - negative, and the function $\delta(x_2)$ has only one zero in this interval.

If x takes a value within the interval [b, a] (Fig. 1), the linearized curve without an inflection point, ie, Linearization is performed by the equation errors at the beginning and end performance. Given the uncertainty in the points $x_2=a\text{-}x$ and $x_2=0$ and making the transformation, we obtain according to the range of $b\ldots a$.

$$\lambda=2/\{1+\left[f_1(a)-f_1(x)\right]/f_1'(x)(a-x)-1\}, \tag{7}$$

$$k=2/\{1/f_1'(x)+(a-x)/\left[f_1(a)-f_1(x)\right]\}. \tag{8}$$

Consider the reduction of x from $x=x_T$. If x is slightly smaller than the x_T, the tangent to the curve at this point will be below the end of the static characteristic. If the line to rotate relative to this point [x, $f_1(x)$] counter-clockwise, the moment crosses the end of the static characteristic $\delta(a\text{-}x)=0$, and at the point $x_2=0$ has to be a certain error. The analysis showed that the growth factor k slope of the error $\delta(0)$ and $\delta(a\text{-}x)$ grows linearly increasing k, but $\delta(0)$ is always greater than $\delta(a\text{-}x)$ on the value of the initial error at the point $x_2=0$, where k corresponds to the passage through the cutting end of the static characteristic. In view of the optimal line is carried out in terms of equality of errors in the points $x_2=0$ and $x_2=n\text{-}x$. With a decrease in the value of x within a certain $\delta(a\text{-}x)$ is decreasing. Therefore, the minimum value of $x=x_н$, where is the linearization according to the conditions will be such that the line which linearized will pass through the end of the static characteristic, i.e. $\delta(a\text{-}x)=0$. Using expressions for the errors in these points and the condition of passing straight through the end of the static characteristic, we can write the condition of the correspondence $x=x_н$

$$1/f_1'(x)+(n-x)/\left[f_1(n)-f_1(x)\right]=2(a-x)/\left[f_1(a)-f_1(x)\right]. \tag{9}$$

If you change the x in the range [0, $x_н$], the function $\delta(x_2)$ has one zero and linearization is performed under the terms of the equality of errors in the points $x_2=0$ and $x_2=n-x$. Given the importance of error for these points, we obtain relations for determining the error of linearization and the slope of the line which linearized as x from 0 to x_T

$$\lambda = 2/\{1+ f_1'(x)(n-x)/[f_1(n)- f_1(x)]\}-1, \tag{10}$$

$$k = 2/\{1/ f_1'(x)+(n-x)/[f_1(n)- f_1(x)]\}. \tag{11}$$

The obtained dependences allow linearized complex structural properties, the nonlinearity of which decreases from the beginning to the end of the measuring range. For similar approach found the conditions of the linearization of complex static characteristics with the growth and low non-linearity. Designed linearization approach allows a better choice of working areas of the static characteristics of media, depending on the conditions of their work.

Carry out the linearization of the static characteristic of passive scanning surface of the open horizontal flow device at steady speeds described by the equation

$$\alpha = 2\arcsin\sqrt{\frac{h}{2l}}, \tag{12}$$

where α - angle of rotation of the scanning element; h - thickness of the material flow; l - length of the scanning element.

Analysis of the equation (12) shows that the non-linearity of the static characteristic will be minimal, and the sensitivity of the maximum with the minimum value of the length of the scanning element. Suppose that the greatest thickness of the material in the stream to be 200 mm, whereas the smallest value of l is also equal to 200 mm. Since static linearity characteristics (12) decreases with increasing input value h, if linearization is necessary to use according to (7) and (8). In this case, point A corresponds to h = 200 mm, therefore, the initial value defined by (12) is $\alpha_{(200)}$=90°. Suppose in this example, the thickness is not less than 140 mm (h = 140 mm), i.e. varies within 140 ... 200 mm. This corresponds to the top band linearization h = 140 mm and

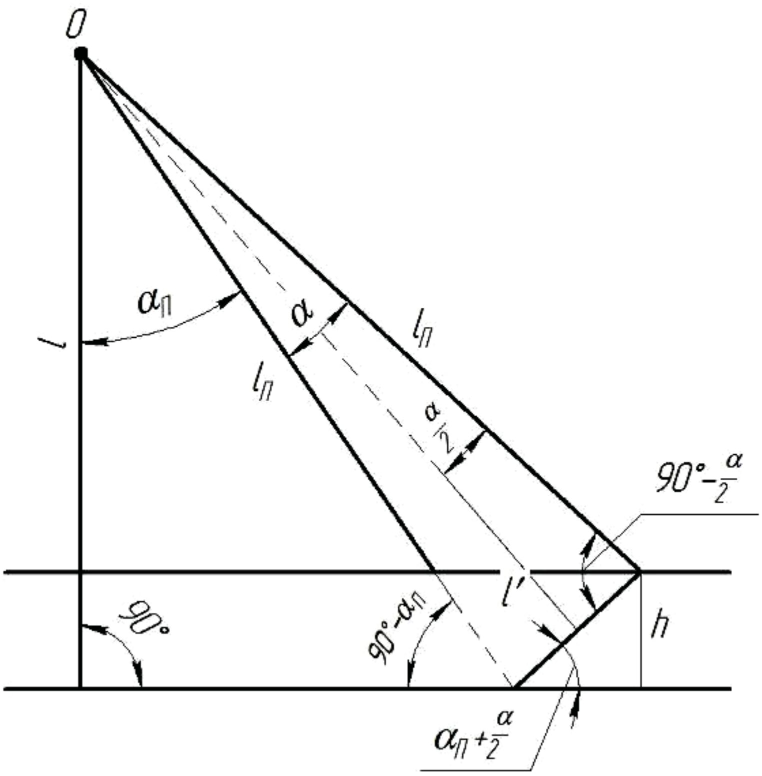

Fig. 2. Measuring the thickness of the layer of material h with an initial angle $\alpha_П$ and extended scanning element

thickness of 0.7 greatest flow. Then, $\alpha_{(140)}$=72,6°. The derivative of equation (12) is

$$\alpha' = 1\Big/\sqrt{h}\sqrt{2l-h}\ . \qquad (13)$$

Its value at h = 140 mm is equal to α'=0,0052 rad / mm or α'=0,2979 deg / mm, and the relative error of linearization defined by the expression (7), will be 1.0134, which corresponds to 1, 34%. For practical purposes, this error is acceptable. The slope of the line which linearized defined by the formula (8) is equal to k = 0,2979 deg / mm. Straight pass through a point with coordinates [h=140 мм; α=72,6°]. For her condition 72,6°=0,2939·140+b, where b=31,45°. Equation the line which linearized will have view

$$\alpha = 0{,}2939 \cdot h + 31{,}45°\ . \qquad (14)$$

Equation (14) allows to determine the thickness h in the flow material changes within 140 mm ... 200 mm with an error not exceeding 1.34%. Measurements are taken at high sensitivity of the device.

Under production conditions may be necessary to measure, for example, a specified error linearization λ=1,34% in the thickness of the material flow on the entire range of 0 ... 200 mm. These conditions can be met, but with some loss of sensitivity of the device. The conditions for the initial angle α_{Π}=72,6° remain, however, change the length of the scanning element and the static characteristic of the device. This case demonstrates the measurement of Figure 2. When the initial rotation angle α_{Π}, which corresponds to the beginning of the range of linearization (h = 140 mm) define a new (initial) length l_{Π} scanning element that corresponds to touch bases transporting means. It will be equal to

$$l_{\Pi} = l/\cos\alpha_{\Pi}\ . \qquad (15)$$

From the construction (Fig. 2) can be written

$$l' = h\Big/\sin\left(\alpha_{\Pi} + \frac{\alpha}{2}\right), \qquad (16)$$

$$l' = 2l_{\Pi}\sin\frac{\alpha}{2}\ . \qquad (17)$$

Equating the right-hand sides of expressions (16) and (17) we obtain

$$h = 2l_{\Pi}\sin\left(\alpha_{\Pi} + \frac{\alpha}{2}\right)\sin\frac{\alpha}{2} \qquad (18)$$

or after the conversion

$$h = l_{\Pi}\left[\cos\alpha_{\Pi} - \cos(\alpha_{\Pi} + \alpha)\right], \qquad (19)$$

whence

$$\alpha_{\Pi} + \alpha = \arccos\left[\cos\alpha_{\Pi} - \frac{h}{l_{\Pi}}\right]. \qquad (20)$$

Static characteristic defined by equation (20) corresponds to the new conditions of measurement, so it must be linearized. The equation (14) for linearization is not suitable because the constant term in this case be equal to 72,6 °, and the coefficient of the variable to be reduced to 1/0, 3 = 3.33 times due to the expansion range from 0.7 .. . 1 0 0 ... 1 or 140 ... 200 mm to 0 ... 200 mm. In this the line which linearized will be described by the equation

$$\alpha = 0{,}0882 \cdot h + 72{,}6° . \qquad (21)$$

For comparison, Table 1 shows values calculated on the static characteristics and the the line which linearized.

Table 1

Comparison of data obtained from the static characteristics and the lines which linearized with different measurement ranges

The height of the layer of material h, mm	Measuring range 140 ... 200 mm		Measuring range 0 ... 200 mm	
	Data on the static characteristic α, °	Data on the line which linearized α, °	Data on the static characteristic α, °	Data the line which linearized α, °
0	0	31,45	72,6	72,6
50	41,34	46,14	76,83	77,01
100	60,0	60,84	81,33	81,42
150	75,34	75,54	85,67	85,83
200	90,0	90,23	90,0	90,24

From the data of Table 1 shows that in the range of 140 ... 200 mm is almost completely identical angles, as shown by the range 150 ... 200mm. At a smaller thickness of the material layer mismatch is quite significant, especially for small values of *h* = 0 ... 50mm. In the second case at the beginning and end of the range angles practically coincide. In the middle of the range of 0 ... 200 mm, deviations, but minor.

Conclusions and direction of future research. Thus, having a complicated static characteristic of an information tool, it can be linearized, in part or in whole, depending on the input value. For this it is necessary to identify the characteristic static characteristic of the point 0, x_T, $x_к$, b, a and justify the range of variation of the argument x. Depending on the range of variation of the input value to the linearization of the proposed use various expressions. The developed approach allows to optimize the static characteristics and design parameters of media regardless of their working conditions, providing a sufficiently high accuracy.

These approaches and the resulting mathematical relationships hold the promise of creating tools scan the surface of open material flows with a certain indicators of accuracy.

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